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Spin Singlet Quantum Hall Effect and Nonabelian Landau-Ginzburg Theory

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ABSTRACT

In this paper we present a theory of Singlet Quantum Hall Effect (SQHE). We show that the Halperin-Haldane SQHE wave function can be written in the form of a product of a wave function for charged semions in a magnetic field and a wave function for the Chiral Spin Liquid of neutral spin- $\frac{1}{2}$ semions. We introduce field-theoretic model in which the electron operators are factorized in terms of charged spinless semions (holons) and neutral spin- $\frac{1}{2}$ semions (spinons). Broken time reversal symmetry and short ranged spin correlations lead to $SU(2)_{k=1}$ Chern-Simons term in Landau-Ginzburg action for SQHE phase. We construct appropriate coherent states for SQHE phase and show the existence of $SU(2)$ valued gauge potential. This potential appears as a result of "spin rigidity" of the ground state against any displacements of nodes of wave function from positions of the particles and reflects the nontrivial monodromy in the presence of these displacements. We argue that topological structure of $SU(2)_{k=1}$ Chern-Simons theory unambiguously dictates *semion* statistics of spinons.

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I. INTRODUCTION.

i) General remarks

It has been assumed from the beginning of the theory of Fractional Quantum Hall Effect (FQHE), that the magnetic field, which has to be strong enough to produce the relevant Landau quantization, leads to large Zeeman splitting. Large body of physical theories of FQHE assumed spins of electrons to be polarized completely (which is equivalent to consideration of spinless electrons in the lowest Landau level).

It has been pointed out first by Halperin¹ that this is not always the case. Zeeman splitting is given by $E_{Zeeman} = g \cdot \mu_B \cdot H$, and Larmour energy is $E_{Larmour} = eH/\hbar c$. The ratio of these two energies depends on the factor $E_{Zeeman}/E_{Larmour} = g \cdot \frac{m^*}{m_0}$, where m^* is the effective mass of electron, and g - is the g -factor. The ratio of m^*/m_0 in the Si/SiO₂ structures is quite small $m^*/m_0 \simeq 0.07$, and g can be as low as $1/4$.

We find, thus, that at least in low enough magnetic fields $B \sim 1$ T, for some materials the ratio $\frac{E_{Zeeman}}{E_{Larmour}} \simeq 0.017$ is quite small. Thus it is a good approximation in this case to neglect Zeeman splitting and consider all states in the Hilbert space of the problem as doubly degenerate due to spin.

Within these assumptions one has to consider the spin unpolarized QHE phase. We will consider below the case of spin singlet QHE phase (SQHE).

Experimentally there is evidence that spin singlet QHE phases are present at some filling factors, see for example.²

In this article we will consider the Landau Ginzburg theory of singlet QHE and how it is connected with nonabelian, namely $SU(2)_{k-1}$ for spin $S = 1/2$, Chern Simons theory a

a natural generalization of the Chern Simons theory for spin polarized case. We will show how the $SU(2)$ valued gauge potential naturally appears in the context of spin coherent states for SQHE³.

But before considering spin unpolarized case we will summarize briefly the most important features of the Landau-Ginzburg theory for spin polarized case.

ii) Spin polarized FQHE and Landau Ginzburg theory

Soon after experimental discovery of the Fractional QHE (FQHE)² Laughlin proposed variational wave function which describes the incompressible electron liquid in 2D in external magnetic field at fractional filling factors, which naturally leads to the “fractional statistics” of the quasiparticles.⁴ The *holomorphic* structure of the Laughlin state relies essentially on the fact that the coordinate space of electron liquid is 2D.

$$\Psi_L(r_i) = \prod_{i < j} (z_i - z_j)^m \exp\left(-\frac{1}{4} |z_i|^2\right) \quad (I.1)$$

The conductivity tensor of this state is

$$\sigma_{xy} = \frac{e^2}{h} \frac{1}{m} \quad (I.2)$$

where m - is an odd integer. The last observation proved to be crucial for the construction of the phenomenology of the fractional Quantum Hall Effect (FQHE).

Namely it has been noticed by Girvin and MacDonald,⁵ and subsequently by others^{6,7} that broken time reversal invariance and parity leads to the possibility for parity noninvariant terms in the Landau-Ginzburg (LG) functional for electrons in FQHE phase. Physics of the Laughlin state Eq. (I.1) is dictated by the fact that in this correlated state each electron at point Z_i is confined with m quanta of magnetic flux $\varphi_o = \frac{hc}{e}$. Effect of the

Chern-Simons term in the LG functional of FQHE is to reinforce the constraint, that *density of particles is proportional to the local flux value of some gauge field*, which we will call the statistical gauge field , A_μ :

$$\varphi^* \varphi - \langle \varphi^* \varphi \rangle = \frac{k}{2\pi} \epsilon_{ij} F^{ij} \quad (I.3)$$

With φ -scalar classical field associated with the order parameter in FQHE.⁷ The reason for introducing extra statistical gauge field is to take into account density fluctuations and associated fluctuations of the phase of the wave function. At the same time the external magnetic field is essentially constant.

The LG functional has the form:^{5,6,7}

$$\begin{aligned} \mathcal{L} = \int d^2x dt \, & \varphi^* (i\partial_0 - e\mathcal{A}_0 - A_0) \varphi - \frac{1}{2m} |(i\partial_i - e\mathcal{A}_i - A_i) \varphi|^2 \\ & + V(\varphi) + \frac{k}{4\pi} A_\mu \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} \end{aligned} \quad (I.4)$$

with $k = \frac{2\pi e^2}{h} \frac{1}{m}$, \mathcal{A}_μ is electromagnetic potential, and $V(\varphi) = -\alpha\varphi^2 + \beta\varphi^4$ is the potential which fixes the amplitude of the order parameter φ . The last term in Lagrangian \mathcal{L} is a Chern Simons terms for the statistical gauge potential A_μ with the group U(1). Variation of \mathcal{L} over electromagnetic potential leads to the expression for the transverse conductivity given by Eq. (I.2). More detailed analysis of the LG theory for FQHE in the spin polarized case can be found in.^{5,6,7,8}

Examining LG theory of spin polarized case we can make two general statements a) The holomorphic structure of the wave function and closely related to its presence of strong magnetic field in the system allows us to write parity and time reversal noninvariant terms, such as Chern Simons for some statistical gauge field.

b) This gauge field obeys the constraint that the *density is proportional to the flux of this gauge field*, like Eq. (1.3). We will argue below that these two statements are also true for the case of LG theory of SQHE. The only essential difference comes from the fact that the gauge group, corresponding to statistical gauge field, will be $SU(2)$. Instead of density operator playing role of the generator of the flux, the spin will be the generator of the spin gauge field flux. In contrast to the $U(1)$ group $SU(2)$ Chern-Simons theory is true topological theory. As a result of quantization of the coefficient in the Chern-Simons term, we will find that the only fractional statistics of excitations for $SU(2)$ at level $k=1$ Chern Simons theory will be *semion* statistics.

II. SPIN SINGLET QUANTUM HALL EFFECT AND LANDAU-GINZBURG THEORY.

i) Halperin-Haldane Wave Function of SQHE and Slave Semion Decomposition.

In this paragraph we consider the physical properties of the singlet Quantum Hall Effect states, given by the Halperin-Haldane wave function^{1,9}:

$$\Psi_m([z_i^+], [z_i^-]) = \prod_{i < j} (z_i^+ - z_j^+)^{m+1} (z_i^- - z_j^-)^{m+1} (z_i^+ - z_j^-)^m e^{-\frac{1}{4} \sum_i |z_i^+|^2 - \frac{1}{4} \sum_i |z_i^-|^2}. \quad (II.1)$$

where the set of coordinates $z_i^+, i = 1, \dots, N$ corresponds to the spin \uparrow electrons, and $z_i^-, i = 1, \dots, N$ corresponds to the spin \downarrow electrons and m is an even integer. In this case $\Psi_m([z_i^+], [z_i^-])$ satisfies the Fock cyclicity condition. In this state, the eigenvalue of the total spin operator is $S = 0$ and the z -component of the spin also has eigenvalue $S_z = 0$. This kind of wave functions naturally appears in the consideration of the spin unpolarized states in the Quantum Hall Effect (QHE) phase.

In contrast to the spin polarized states, in this case we need to describe the charge sector of the SQHE phase as well as the spin sector. By inspecting the structure of this wave function one finds that it has the simple but very important property that the spin and charge degrees of freedom are factorized. The total wave function $\Psi_m([z_i^+], [z_i^-])$ can be written as a product of the charge wave function $\Psi_m^{(1)}([z_i^+], [z_i^-])$ and spin wave function $\Psi_m^{(2)}([z_i^+], [z_i^-])$. Below we will discuss the properties of the charge and spin wave functions separately. At the end we will put them together again by imposing the constraint that the positions of the charges coincides with those of the spins. This property is strongly reminiscent of the charge and spin separation present in models of Strongly Correlated Electron systems in the context of theories of high temperature superconductors¹⁰.

The wave function is factorized in the following manner¹¹:

$$\Psi_m([z_i^+], [z_i^-]) = \Psi^{(2)}([z_i^+], [z_i^-]) \Psi_m^{(1)}([z_i^+], [z_i^-]). \quad (II.2)$$

with

$$\Psi_m^{(1)}([z_i^+], [z_i^-]) = \prod_{i < j} (z_i^+ - z_j^+)^{m+1/2} (z_i^- - z_j^-)^{m+1/2} (z_i^+ - z_i^-)^{m+1/2} e^{-\frac{1}{4} \sum_i |z_i^+|^2 - \frac{1}{4} \sum_i |z_i^-|^2}. \quad (II.3)$$

$$\Psi^{(2)}([z_i^+], [z_i^-]) = \prod_{i < j} (z_i^+ - z_j^+)^{1/2} (z_i^- - z_j^-)^{1/2} (z_i^+ - z_i^-)^{-1/2}. \quad (II.4)$$

Why does this decomposition make sense?. The plasma analogy, when applied to $\Psi_m^{(1)}([z_i^+], [z_i^-])$, shows that this state is described by a one component plasma, in which the particles at points z_i^+ and z_i^- have equal charge:

$$|\Psi_m^{(1)}([z_i^+], [z_i^-])|^2 = \exp((2m+1)(\sum_{i < j} \ln |z_i^+ - z_j^+| + \sum_{i < j} \ln |z_i^- - z_j^-| + \sum_{i,j} \ln |z_i^+ - z_j^-| - 1/2 \sum_i |z_i^+|^2 - 1/2 \sum_i |z_i^-|^2)). \quad (II.5)$$

We regard $\Psi_m^{(1)}([z_i^+], [z_i^-])$ as the wave function for the charge degrees of freedom.

If we apply the same plasma analogy to the wave function $\Psi^{(2)}([z_i^+], [z_i^-])$ we get¹¹:

$$|\Psi^{(2)}([z_i^+], [z_i^-])|^2 = \exp(\sum_{i < j} \ln |z_i^+ - z_j^+| + \sum_{i < j} \ln |z_i^- - z_j^-| - \sum_{i,j} \ln |z_i^+ - z_j^-|). \quad (II.6)$$

and we can easily see that $\Psi^{(2)}([z_i^+], [z_i^-])$ corresponds to a two-component plasma, where the effective charge of the particles q is given by the spin projection $q = 2s_z = \pm 1$. It is natural to consider $\Psi^{(2)}([z_i^+], [z_i^-])$ as the wave function of the spin degrees of freedom.

We will show below that $\Psi_m^{(1)}([z_i^+], [z_i^-])$ can be regarded as a wave function for semions in an external magnetic field. From eq (II.3) we conclude that, for any m , $\Psi_m^{(1)}([z_i^+], [z_i^-])$ describes particles with semion statistics: any exchange of two of them leads to a change of phase of $\pi(m + 1/2)$ and, if m is even, this particles are semions.

From the same considerations it follows that $\Psi^{(2)}([z_i^+], [z_i^-])$ represents a two-component semion gas. The sign of the spin projection s_z determines the effective phase change in any interchange of two particles $q_1 q_2 \pi/2$, where q_1, q_2 are ± 1 for spin \uparrow, \downarrow . This model with two component semions was considered in ^{12,11}. In particular, Girvin et al. ¹¹ have pointed out that the state described by the wave function $\Psi^{(2)}([z_i^+], [z_i^-])$ is a local spin singlet due to the plasma screening of any charge.

The decomposition of eq (II.2) can be represented in terms of the slave *semion* operators:

$$\psi_\sigma(r) = \varphi(r)\xi_\sigma(r). \quad (II.7)$$

where $\psi_\sigma(r)$ is the electron operator, $\varphi(r)$ is a charge e spinless semion operator, ξ_σ is a spin $1/2$ charge-neutral semion operator, σ is a spin index, and we assume that $[\varphi(r), \xi(r)] = 0$.

In principle this decomposition is neither better nor worse than any other slave boson or slave fermion factorization, like the ones that are commonly used in theories of strongly correlated systems. The choice of any particular decomposition of the initial electron operator is purely a matter of convenience. Our choice is motivated by the simplicity of the physical picture that we get in the end.

In Mott-Hubbard insulators, the strong correlations force the constraint of single particle occupancy. In the case of the SQHE, the origin of the strong correlations is the drastic reduction of phase space due to the presence of a strong magnetic field: the kinetic energy is quenched and the interactions dominate. In close analogy with the Mott-Hubbard problem, we argue that in the Singlet Quantum Hall Effect the spin and charge degrees of freedom are separated in the sense of the decomposition of eq (II.7). Here too, a gauge symmetry arises as a result of this factorization. This gauge symmetry means that the *relative* phase between charge and spin states is not a physically observable degree of freedom. The SQHE wave function is a singlet under this gauge symmetry. However, the

decomposition eq (II.7) requires that the entire spectrum of states must be singlets under this gauge symmetry. Given the close analogy with the Mott-Hubbard problem, we will refer to this symmetry as the RVB gauge symmetry. The presence of this RVB gauge symmetry gives rise to an RVB gauge field which puts the charge and spin semions together to form the allowed physical states. Thus, although the wave functions of all the states can be factorized as a product of a *charge* and *spin* wave functions, there is no separation of spin and charge in this system. In consequence, the system has a gap to *all* excitations and it is *incompressible*. The factorized form of the SQHE wave function, Eq (II.2), appears to suggest that there may be a gapless neutral spin excitation which would lead to *compressibility*. Because the RVB gauge charge is confined, these excitations are not a part of the physical spectrum. It is important to stress that the incompressibility results entirely from the charge sector.

Perhaps the simplest way to see this is to consider the wave function of quasiparticle (qp) in the first quantized representation, as it has been done in ⁹. for example, for the qp of spin 1/2 with $s_z = -1/2$ at point z_0 :

$$\Psi_{z_0,1}([z_i^+], [z_i^-]) = \prod_i (z_i^+ - z_0) \Psi_m([z_i^+], [z_i^-]) \quad (II.8)$$

The form of this wave function indicates that the creation of the qp is equivalent to the creation of the extra zero at point z_0 for the wave function of the particles with the spin $s_z = +1/2$ projection. By using the plasma analogy it is easy to conclude that this zero is equivalent to the qp of spin $s_z = -1/2$ with charge $e = \frac{1}{2m+1}$.

Now we will explicitly show that the wave function of the qp in the SQHE can be represented as a composite excitation of neutral spinon with $s = 1/2$ and of the spinless holon with charge $e = \frac{1}{2m+1}$. We can rewrite $\Psi_{z_0,1}$ as :

$$\begin{aligned} \Psi_{z_0,1}([z_i^+], [z_i^-]) &= \prod_i (z_i^+ - z_0)^{1/2} (z_i^- - z_0)^{1/2} \Psi_m^{(1)}([z_i^+], [z_i^-]) \\ &\quad \prod_i (z_i^+ - z_0)^{1/2} (z_i^- - z_0)^{-1/2} \Psi_m^{(2)}([z_i^+], [z_i^-]) \end{aligned} \quad (II.9)$$

The first product $\Psi_{z_0}^{(1)} = \prod_i (z_i^+ - z_0)^{1/2} (z_i^- - z_0)^{1/2} \Psi_m^{(1)}([z_i^-], [z_i^+])$ is nothing more than the holon excitation in the one component plasma, corresponding to the $\Psi_m^{(1)}([z_i^+], [z_i^-])$. From this follows that the effective charge of the holon is $e = \frac{1/2}{m+1/2} = \frac{1}{2m+1}$. The second product $\Psi_{z_0}^{(2)} = \prod_i (z_i^+ - z_0)^{1/2} (z_i^- - z_0)^{-1/2} \Psi^{(2)}([z_i^+], [z_i^-])$ is the spinon excitation, corresponding to the extra spin $s_z = -1/2$ excitation, created at point z_0 .

There is an apparent problem with the identification of the sign of the spin projection for the excitation $\Psi_{z_0}^{(2)}$. By the plasma analogy the fictitious spin 1/2 at point z_0 has the same projection as the spinons at points z_i , i.e. $s_z = +1/2$. But then, due to the plasma screening in the two component plasma, the real spinons will screen out this fictitious spin, thus creating the $s_z = -1/2$ cloud of real spinons, centered at point z_0 . This is precisely the reason why the spin projection of the excitation $\Psi_{z_0}^{(2)}$ is down.

Once this confusing point has been clarified, we come to the statement that the spin 1/2 charge $e = \frac{1}{2m+1}$ qp can be represented as a product of the spinon and holon qp created at the same point z_0 :

$$\Psi_{z_0} = \Psi_{z_0}^{(1)} \Psi_{z_0}^{(2)} \quad (II.10)$$

The eq(II.10) is a decomposition in Eq.(II.7) written in the first quantized representation.

We find that the slave semion decomposition (II.7) for the SQHE is valid not only in the ground state but for the qp excitations as well. Clearly the argument given above can be generalized trivially for the case of n qp. The fact that we need to put our spinon and holon on the same place explicitly indicates that these excitations with opposite RVB charge are confined to form an RVB neutral object, only allowed as the physical state³.

Thus we showed that the slave semion decomposition eq.(II.7) is quite natural way to distinguish the physics in the charge and spin sector of SQHE. This *factorization* (but not *separation*) can be observed for any state in the Hilbert space of SQHE.

ii) Coherent states for SQHE

In this section we will introduce coherent states for SQHE. Originally coherent states were introduced by Read⁷ for FQHE in the spin polarized case. The generalization of this construction towards spin unpolarized case is straightforward. In this construction we will use analogy with the coherent states for superconductors.

Suppose we have a system, in which some composite operator, involving few particle operators acquire the nonzero expectation value. An example of such an operator is a superconducting order parameter

$$\Delta = \langle N | \psi_{\alpha k}^+ \psi_{-\alpha k}^+ | N + 2 \rangle \quad (II.11)$$

where $\psi_{\alpha k}^+$ is the single particle operator with spin α and momentum \vec{k} , and $|N\rangle$ is the wavefunction of superconductor with N particles.

Clearly, the two particle operator such as in Eq. (II.11) can not have nonzero expectation value in the state with the fixed number of particles $|N\rangle$, $\langle N | \psi_{\alpha k}^+ \psi_{-\alpha k}^+ | N \rangle \equiv 0$, because this object is not gauge invariant under global $U(1)$ gauge transformations. In thermodynamic limit we usually consider the system with fixed chemical potential and indefinite number of particles which allows the operator to have a nonzero expectation value. This kind of states allows us to get nonzero expectation value for the two particle operator. The phase of the order parameter Δ has to be well defined in superconductor, and taking into account that density operator $\hat{n}_k = \psi_{\alpha k}^+ \psi_{\alpha k}$ and phase φ_k are canonically conjugated variables $[\hat{n}_k, \varphi_{k'}] = i\delta(k - k')$, we find that the superposition of states with indefinite number of particles but with fixed phase are natural for considering superconductors.

These coherent states

$$|\theta\rangle = \sum_{N=1}^{\infty} \frac{1}{N!} e^{iN\theta} |N\rangle, \quad (II.12)$$

where θ is the phase, β_N is some weight which is peaked around macroscopical value $N = \bar{N}$ with variance $\Delta N \sim \bar{N}^{1/2}$. In this basis one easily find that the order parameter becomes a classical field:

$$\Delta = \langle \theta | \psi_{k\alpha}^+ \psi_{-k\beta}^+ | \theta \rangle = |\Delta_o| e^{i\eta} \quad (II.13)$$

with the well defined amplitude $|\Delta_o|$ and phase η .

From this transparent example we conclude that if the order parameter as an operator involves few particle operators the appropriate basis for consideration of this phase are the coherent states which are coherent superposition of states with different number of particles.

Application of coherent states for construction of the LG theory of polarized FQHE has been done by N. Read,⁷ see also.¹³ Here we will follow these ideas to construct the coherent states for spin unpolarized QHE.

Define states $|N_+, N_- \rangle$ as:

$$|N_+, N_- \rangle \equiv \Psi_m([z_i^+], [z_i^-]) \quad (II.14)$$

where N_{\pm} is the number of particles with up(down) spin. Introduce the coherent states:

$$|\theta_+, \theta_- \rangle = \sum_{N_{\pm}} \beta_{N_+} \beta_{N_-} e^{-iN_+\theta_+ - iN_-\theta_-} |N_+, N_- \rangle \quad (II.15)$$

where $\beta_{N_{\pm}}$ are some weights with $\langle N_{\pm} \rangle = \bar{N}$, and some variance, $\Delta N_{\pm} \sim \bar{N}^{1/2}$. This state is with undefined S_z and undefined number of particles. The following composite operator acquires the nonzero expectation value in the $|\theta_+, \theta_- \rangle$ state¹³:

$$\Lambda_+(z) = \psi_+^\dagger(z) U_+^{m+1}(z) U_-^m(z) \quad (II.16)$$

$$\langle \theta_+, \theta_- | \Lambda_+ | \theta_+, \theta_- \rangle = \text{const} \quad (II.16b)$$

where $U_{\pm}(z)$ is the flux operator, which produces a node in the wave function $|\theta_+, \theta_- \rangle$:

$$U_{\pm}(z) = \prod_i (z - z_i^{\pm}) \quad (II.17)$$

And state $|\theta_+, \theta_- \rangle$ is simply the condensate of the composite operators:

$$|\theta_+, \theta_- \rangle \sim \sum_{N_{\pm}} \beta_{N_+} \beta_{N_-} \left(\int d^2 z^+ \Lambda_+(z^+) \right)^{N_+} \left(\int d^2 z^- \Lambda_-(z^-) \right)^{N_-} e^{iN_{\pm}\theta_{\pm}} |0 \rangle \quad (II.18)$$

Eqs. (II.16)-(II.18) are just the mathematical expression of the physically transparent fact that in the Halperin-Haldane state the electrons of spin up and down are confined with the zeros of the wave function. For example each electron of spin up is confined with the $(m+1)$ st power of zero in the wave function for all other spin up electrons and m -st power for electrons of spin down.

As we are mainly concerned with spin dynamics of SQHE, we consider coherent states and the appropriate order parameter for the spin wave function $\psi^{(2)}([z_i^+], [z_i^-])$ in Eq. (II.4). It has been argued¹⁴ that this wave function describes the spin 1/2 Chiral Spin Liquid (CSL) state. It follows from this that the spin dynamics of SQHE and spin dynamics of CSL phase are closely related. However there is one principal difference between spin excitations allowed in SQHE and CSL: the only spin excitations allowed in the bulk of SQHE are gaped spin $S=1$ spin waves, while there are spin 1/2 spinons in the CSL state. This difference comes from the fact that spinons in the bulk of SQHE sample are confined because of analyticity of the wave function, as we mentioned earlier.

The composite operators which condense in the CSL state are

$$\Lambda_+^{CSL}(z) = \psi_+^+(z) U_+^{-1/2}(z) U_-^{-1/2}(z) \quad (II.19)$$

Coherent states natural for CSL are given by Eq. (II.18) with obvious substitution $\Lambda_\pm \rightarrow \Lambda_\pm^{CSL}$.

The composite operator Λ_\pm^{CSL} describes the condensation of the flux $\pm\pi$ on the particles with spin $S_z = \pm$. Half flux condensation implies that semion statistics of excitations should be expected in this state, and indeed as it is known that spinons are fractional statistics excitations in this state.¹⁴

Using these facts we are now ready to construct the LG theory of SQHE phase.

iii) $SU(2)_{k=1}$ Chern Simons Theory as a LG Functional for Singlet QHE.

Below we will consider only spin aspect of the LG theory of SQHE, and thus only the neutral excitations will be considered. Because of decomposition Eq. (II.2), the charge sector can be treated analogously to the derivation of LG theory for spin polarized FQHE.

Due to the charge-spin factorization in the Halperin-Haldane state, we will use composite operator factorization

$$\Lambda_\pm(z) = \Lambda_\pm^{CSL}(z) \cdot \Lambda^{charge}(z) \quad (II.20)$$

with obvious form for $\Lambda^{charge}(z)$ which is independent on spin indexes, and analogously for coherent states:

$$|\theta_+, \theta\rangle = |\theta_+, \theta\rangle_{spin} |\theta_+, \theta\rangle_{charge} \quad (II.21)$$

Obviously the operator, relevant for spin dynamics is $\Lambda_\pm^{CSL}(z)$ and the wavefunction containing all information about spin configurations of electrons is $|\theta_+, \theta\rangle_{spin}$, defined a

$$\begin{aligned}
|\theta_+, \theta_-\rangle_{spin} &= \sum_{N_+, N_-} \beta_{N_+} \beta_{N_-} e^{iN_+ \theta_+} \\
&\times \left(\int d^2 z_+ \Lambda_+^{CSL}(z_+) \right)^{N_+} \left(\int d^2 z_- \Lambda_-^{CSL}(z_-) \right)^{N_-} |0\rangle \quad (II.22)
\end{aligned}$$

with the same notations used as was used in the definition of $|\theta_+, \theta_-\rangle$ Eq. (II.15). In what follows we will drop the “CSL” from the spin composite operator $\Lambda_{\pm}^{CSL}(z)$ and “spin” from the spin part of the wavefunction $|\theta_+, \theta_-\rangle_{spin}$.

Crucial object in deriving the LG functional is the gauge potential, which appears as a result of displacement of zero of the wavefunction $\psi^{CSL}([z_i^+], [z_i^-])$ from the position of electron to which this zero is confined. Namely, consider the following operator:

$$\Lambda_{\mu}(z, z') = \prod_{\mu'} \psi_{\mu}^+(z) \mathcal{L}_{\mu'}^{1/2\mu\cdot\mu'}(z') \quad (II.23)$$

where $\mu, \mu' = \pm$, and at $z = z'$ this operator is just the $\Lambda_{\mu}(z)$, discussed above. Below we will assume that operators $\Lambda_{\mu}(z, z')$ are normalized by the factor

$N_{\mu} = \langle \theta_+, \theta_- | \prod_{\mu'} \mathcal{L}_{\mu'}^{1/2\mu\cdot\mu'}(z) | \theta_+, \theta_- \rangle^{-1}$ what will be taken into account in the expansion of the composite operator. The displacement $z = z'$ between the point at which the particle was created and the point at which the zeros of wave function are located may lead to nontrivial monodromy properties of the wave function in the presence of such displacements. Physically this nontrivial monodromy of particle wave function around closed contour C , enclosing such displacements, leads to a frustration of the wavefunction. This in turn leads to the increase of energy. The system prefers the ground state in which the zeros of wavefunction are confined to the positions of the particles.⁵ The gauge potential which reflects nontrivial monodromy of probe particle in the nonhomogeneous case in spin polarized FQHE, see Eq. (I.3), appears naturally in this analysis.

The difference between polarized FQHE and SQHE is that in SQHE phase pure spin distortions can produce gauge potential, even if the charge fluctuations do not lead to any U(1) gauge potential as in Eq. (I.3). This concept of binding zeros of spin wave function with the particles was called “spin rigidity” in the case of CSL to stress the topological effect caused by displacements between zeros of wave function and positions of particles.¹⁵

Here we shall see that the same “spin rigidity” of the SQHE ground state in the spin sector leads to SU(2) valued gauge potential $\hat{A}_x = i \cdot \hat{A}_x^i \cdot \sigma_{\alpha\beta}^i$, $\sigma_{\alpha\beta}^i$ are Pauli matrices. This gauge potential measures the nontriviality of monodromy of spin wave function. Define $\hat{A}_\pm = \hat{A}_x \pm i\hat{A}_y$ and:

$$iA_-^{\nu\mu} = \lambda \int \frac{d^2 z'}{z' - z} \langle \Lambda_\mu^+(z', z) \wedge_\nu(z' z) \rangle \quad (II.24)$$

where λ is the coefficient to be defined later. Taking into account Eq. (II.23), and approximating $\langle \Lambda_\mu^+ \wedge_\nu \rangle \cong (-1) \langle \psi_\nu^+(z') \psi_\mu(z') \rangle$ we find:

$$i \partial_z A_- = \lambda \pi \langle \psi_\nu^+(z) \psi_\mu(z) \rangle \quad (II.25)$$

or in terms of spin components:

$$i \partial_z A_-^i = \lambda \pi \langle S^i(z) \rangle \quad (II.26)$$

As we mentioned, the ground state expectation value of spin operator is zero. Any spin excitation, however, produce the gauge potential A^i . For example, for spin 1/2 quasihole in Halperin-Haldane state $\langle S^z \rangle = \delta(z - z_0)$ will lead to a gauge potential of a point line source:

$$A^z(z') = \frac{\lambda}{2} \int \frac{d^2 z}{z - z'} \langle \psi^\dagger(z) \sigma^z \psi(z) \rangle$$

$$= -\frac{\lambda}{2} \frac{1}{z_o - z'} \quad (II.27)$$

The value of λ is fixed by the requirement to be consistent with semion statistics of spin 1/2 excitations (neglecting the phase coming from charge sector) and gives $\lambda=1$. From Eq. (II.24) it follows that in SQHE phase the displacement between zeros of wave function and positions of particles leads to a nonlocal effect, revealed by effective gauge potential. Assuming that scalar interactions in the system are short ranged, we can write down the local effective LG action whose variation leads to constraint Eq. (II.24 – II.26):

$$\begin{aligned} S = & \int d^2x dt \, \Lambda_\mu^+ (i \partial_o 1^{\mu\nu} - A^{\mu\nu}) \Lambda_\nu + \frac{1}{4\pi} \text{Tr} \hat{A}_i \partial_j \hat{A}_k \epsilon^{ijk} \\ & + V(\Lambda^+ \Lambda) + \frac{1}{2M} |(i \partial_i 1^{\mu\nu} - A_i^{\mu\nu}) \Lambda_\nu|^2 \end{aligned} \quad (II.28)$$

Where we also take into account special gradients of the order parameter $\Lambda_\mu(z)$ defined in Eq. (II.19); potential $V(\Lambda^+ \Lambda)$ provides the fixed amplitude of the order parameter. The most nontrivial part of the effective LG action is the $\text{Tr} \hat{A}_i \partial_j \hat{A}_k \epsilon^{ijk}$ term, which is recognized as a gradient part of the $\text{SU}(2)_{k=1}$ Chern-Simons term:

$$\mathcal{L} = \int d^2x dt \, \frac{k}{4\pi} \text{Tr}(\hat{A}_i \partial_j \hat{A}_k + 2/3 \hat{A}_i \hat{A}_j \hat{A}_k) \epsilon^{ijk} \quad (II.29)$$

at $k \equiv 1$ for our case (the subscript k in $\text{SU}(2)_k$ means precisely the coefficient in front of Chern-Simons term).

It is reasonable to argue that because of local spin correlations in SQHE state the true $\text{SU}(2)$ rotational invariance should be observed. Although in Section i) identify the spin-1 particles with flux $\pm \pi$ in state $\Psi^{(2)}([z_i^+], [z_i^-])$ this identification requires the spin quantization axis to be fixed explicitly. This is the “abelian” way to incorporate spin quantum numbers of electrons into the wave function.

In this procedure the single particle states are described in terms of the spin projection on the z-axis, and for simplicity, this axis is assumed to be in the same direction everywhere. Thus, we deal only with the $U(1)$ diagonal subgroup of the full $SU(2)$ spin group. Also, the plasma analogy for $\Psi^{(2)}([z_i^+], [z_i^-])$ leads to the correspondence with the two component plasma with effective charge $q = +q_0$ for $\text{spin}\uparrow$ and $q = -q_0$ for $\text{spin}\downarrow$ particles. This analogy suggests that we should attach different fluxes to particles with opposite spin and deal with them in much the same way as we did with the charge sector in section II.

However, there is a problem with this approach. So far there is no spin anisotropy in this state since we have neglected the Zeeman term in the consideration of the SQHE⁹. The “abelian” approach breaks the $SU(2)$ spin symmetry from the outset. Its recovery is a highly non-trivial matter. In principle one has to be able to formulate the SQHE wave function while keeping the full $SU(2)$ invariance and to allow for a quantization axis that is varying in space. Girvin et al.¹¹ have pointed out that $\Psi^{(2)}([z_i^+], [z_i^-])$ leads to a partition function for a two-component plasma and that any extra charge = spin is screened. The screening in the two component anyon gas, in the context of the spin coupled to a gauge field, was found in reference 12. Thus, what is needed is a procedure to attach different fluxes to particles with \uparrow and \downarrow spins in a manner that is compatible with the $SU(2)$ spin symmetry. Fortunately such an approach does exist: it is the non-abelian $SU(2)$ CS theory. A non-abelian CS term, much like the abelian CS theory used in the description of the spin polarized QHE^{5,6,7,16}, attaches fluxes to particles. But, unlike the “abelian” approach mentioned above, the non-abelian CS theory is invariant under $SU(2)$ rotations of the spin. Furthermore, this invariance is local and the theory is a gauge theory. It turns out that the CS theory represents the only possible local way to attach particles to $SU(2)$ fluxes. Below we will follow this second way in considering the spin wave function.

Consider the set of coordinates $[z_i^+], [z_i^-]$ of a set of some spinors with the spin up components, located at points $[z_i^+]$, and spin down at points $[z_i^-]$. The points $[z_i^+], [z_i^-]$

will be regarded as the positions of sources of an $SU(2)$ field Λ_μ , taken in the fundamental representation. It corresponds to the spin 1/2 of the electrons, constituting the QHE state. The Lagrangian $\mathcal{L}_{\text{spin}}$ of the spin sector is given by Eq.(II.28) with the full non-abelian Chern-Simons term.

The points at which the excitations are located are the sources for the gauge field. As it can be seen from the variation of the Lagrangian (II.28) over A_0^a :

$$-\frac{\delta \mathcal{L}_{\text{spin}}}{\delta A_0^a} = \Lambda^+ \sigma^a \wedge + \frac{k}{\pi} F_{xy}^a = 0. \quad (II.30)$$

The strength of the gauge field is given by $F_{xy}^a = \partial_x A_y^a - \partial_y A_x^a + [A_x, A_y]^a$. Let us assume that the particles have a mass m . The path-integral representation of a matrix element of the evolution operator is given as a sum over all possible particle trajectories and gauge field histories. The constraint of Eq (II.30) requires that each term in this amplitude should contain a factor representing a path-ordered exponential of the $SU(2)$ gauge field along each particle trajectory. These path-ordered exponentials are usually referred to as Wilson lines. In first quantization, the time evolution during the time interval t of the heavy sources will be given by the amplitude:

$$\Psi([z_i'^+], [z_i'^-], t) = \sum_{\text{Paths}} e^{-i \int dt (\sum_i m/2 |dz_i^+/dt|^2 + \sum_i m/2 |dz_i^-/dt|^2)} \int D[A] \odot_{i,j} W_i(z_i'^+, z_i^+) W_j(z_j'^-, z_j^-) e^{ik \int d^2x dt \mathcal{L}_{CS}} \Psi([z_i^+], [z_i^-], 0) \quad (II.31)$$

where z'^+, z'^- are the set of final positions of the sources, and

$$W_i(z_i', z_i) = [P e^{i \int_{z_i}^{z_i'} A_\mu dx^\mu}], \quad (II.32)$$

are Wilson lines evaluated on the 3 dimensional paths from z_i to z_i' . We will consider the 2-D disc geometry pierced by the Wilson lines. The coordinate space is $D \times R$, where R is the time. The integral in the exponent in $W_i(z_i', z_i)$ is the quasiclassical expression for the spin-current-gauge potential coupling $\int A_\mu j_\mu^a d^2x dt$, assuming that $j_\mu^a = \sigma^a \frac{dx^\mu}{dt} \delta(x - x_I(t))$ and $x_I(t)$ parametrizes the quasiclassical path of the particle.

The CS action for the gauge field leads to the effective semion statistics of Wilson lines. Let us fix two Wilson lines, corresponding, for example to particles at z_1 and η_1 . And let us consider two processes which represent evolutions with the same final state and only differ by the presence of an extra knot in their histories given by $W(z_i'^+, z_1^+), W(z_1'^-, z_1^-)$. Then the final amplitudes $\mathcal{V}([z_i'^+], [z_i'^-])$ will gain different phases in these processes. One can find¹⁷, that the amplitudes are related by

$$\Psi_{knotted}([z_i'^+], [z_i'^-]) = \exp(i\gamma) \Psi_{unknotted}([z_i'^+], [z_i'^-]) \quad (II.33)$$

where γ is the *conformal weight* of the primary field for the $SU(2)$ level k group, and is given by

$$\gamma = \frac{4\pi j(j+1)}{k+2} \quad (II.34)$$

In our case, $k = 1, j = \frac{1}{2}$, the phase difference between two configurations is π which corresponds to a phase of $\pi/2$ per particle. If we assume that the evolution between two configurations is adiabatic, the kinetic energy does not modify the value of γ because it is quadratic in time derivative. The only contribution to the phase comes from the CS action and it leads to the semion statistics of the excitations, exhibited in the spin wave function $\Psi^{(2)}([z_i^+], [z_i^-])$ ¹⁴.

III. CONCLUSION

In this paper the theory of SQHE phase was presented. We considered the charge spin factorization in the Halperin Haldane state and argue that the Halperin Haldane state variational wave function can be written in the form of product of two wave functions: one, $\Psi_m^{(1)}([z_i^+], [z_i^-])$ corresponds to charged spinless semions in external magnetic field and the other — spin wave function $\Psi^{(2)}([z_i^+], [z_i^-])$ is the wave function of spin 1/2 neutral

semions. Because all states in the Hilbert space of the problem can be represented as a direct product of charge and spin contribution we argue that LG theory of SQHE phase can be written as the theory for composite order parameter $\Lambda_{\pm}(z)$ from Eq. (II.20). In our derivation of the LG theory we concentrate on the spin sector. The parts of LG action for charge sector can be derived, following the derivation of LG theory for spin polarized FQHE.

We construct the coherent states for SQHE phase which are analogous to the coherent states for polarized FQHE phase, and describe the state with undefined spin projection and undefined number of particles.

Because of the “spin rigidity” of SQHE state, the modes of spin wave function $\Psi^{(2)}([z_i^+], [z_i^-])$ are confined to the positions of the particles. Moreover we find that any displacements of these modes from positions of the particles, described by nonlocal composite operator $\Lambda_{\mu}(z', z) = \psi_{\mu}^+(z') \prod_{\mu'} U_{\mu'}^{1/2\mu\cdot\mu'}(z)$ leads to nontrivial monodromy of the wave function around closed contour, enclosing such a displacement. The natural measurement of this monodromy of spin wave function is the $SU(2)$ valued gauge potential \hat{A}_i with the flux F_{xy}^i proportional to the noncompensated spin density $\langle \hat{S}^i \rangle$. Although we were not able to reproduce full $SU(2)$ invariant topological term, we argue that because of local $SU(2)$ invariance in SQHE phase, the spin part of LG action contains $SU(2)_{k=1}$ Chern Simons term. We also find that the topological structure of the Chern-Simons theory leads unambiguously to semion statistics of excitations in the spin sector of SQHE. However, these excitations are not physically relevant, because in the bulk of SQHE phase spinons are confined with holons in order to have trivial monodromy for Halperin-Haldane wave function, written in terms of electron coordinates. It has been argued in¹⁸, that $SU(2)_{k=1}$ Kac-Moody algebra, which describe the edge of SQHE sample leads to the neutral spinon excitations as part of the Hilbert space of the edge.

This consideration of $S=1/2$ SQHE state is also useful in revealing the connection

between conformed field theory and different phases of FQHE.^{13,3} For example using these results we can show that i) there is a $S=1$ Singlet QHE variational wave function, ii) this wave function is given by conformal block of $SU(2)_{k=2}$ Chern-Simons theory, iii) it supports nonabelian excitations with fractional charge and spin $1/2$.¹⁹

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